OSU-TA-6/97
April 1997

Temperature Fluctuations and Abundances in H II Galaxies

Gary Steigman[1], Sueli M. Viegas[2] and Ruth Gruenwald[2]

[1]*Departments of Physics and Astronomy, The Ohio State University, Columbus, OH 43210, USA*

[2]*Instituto Astronômico e Geofísico, Universidade de São Paulo, São Paulo, S.P. 04301-904, BRASIL*

Abstract

There is evidence for temperature fluctuations in Planetary Nebulae and in Galactic H II regions. If such fluctuations occur in the low-metallicity, extragalactic H II regions used to probe the primordial helium abundance, the derived ^{4}He mass fraction, Y_P, could be systematically different from the true primordial value. For cooler, mainly high-metallicity H II regions the derived helium abundance may be nearly unchanged but the oxygen abundance could have been seriously underestimated. For hotter, mainly low-metallicity H II regions the oxygen abundance is likely accurate but the helium abundance could be underestimated. The net effect is to tilt the Y vs. Z relation, making it flatter and resulting in a higher inferred Y_P. Although this effect could be large, there are no data which allow us to estimate the size of the temperature fluctuations for the extragalactic H II regions. Therefore, we have explored this effect via Monte Carlos in which the abundances derived from a fiducial data set are modified by ΔT chosen from a distribution with $0 \leq \Delta T \leq \Delta T_{max}$ where ΔT_{max} is varied from 500K to 4000K. It is interesting that although this effect shifts the locations of the H II regions in Y vs. O/H plane, it does not introduce any significant additional dispersion.

1 Introduction

The primordial abundance of ^{4}He is key to testing the consistency of the standard hot big bang model of cosmology and to using primordial nucleosynthesis as a probe of particle physics (Steigman, Schramm & Gunn 1977). The availability of large numbers of carefully observed, low-metallicity H II regions has permitted estimates of the primordial helium mass fraction, $Y_{\rm P}$, whose statistical uncertainties are very small, $\approx 1\%$ (see, Olive & Steigman 1995 (OS), Olive, Skillman & Steigman 1997 (OSS) and references therein). However, there remains the possibility that in the process of using the observational data to derive the abundances, contamination by unacknowledged systematic uncertainties has biased the inferred value of $Y_{\rm P}$. Observers have identified many potential sources of systematic uncertainties (Davidson & Kinman 1985; Pagel *et al.* 1992 (PSTE); Skillman *et al.* 1994; Izotov, Thuan & Lipovetsky 1994, 1997 (ITL); Peimbert 1996) and, where possible, have designed their observing programs to minimize such uncertainties and/or to account for them. It is expected that the contributions from many of the potential sources of systematic uncertainty (if present at all) would vary from H II region to H II region and from observer (telescope/detector combination) to observer, introducing along with a systematic offset in the derived value of $Y_{\rm P}$, an accompanying dispersion in the helium data. More insidious would be systematic errors in, for example, the atomic physics used to convert the observed equivalent widths to abundances, since an offset from the "true" value of $Y_{\rm P}$ would not be accompanied by an enhanced dispersion in the fits to the data (e.g., Y vs. O/H or the weighted mean of Y for the lowest metallicity H II regions; see OSS). In this paper we explore one potential source of systematic uncertainty in deriving abundances from emission-line data for extragalactic, H II regions: temperature fluctuations.

The empirical method to derive chemical abundances in an emitting gas has been used widely since it was first proposed by Peimbert and Costero (1969). The electron density n_e and the gas temperature T are obtained from emission-line intensity ratios and are used to calculate the line-emissivity which, along with the observed line-intensities, provide the fractional abundance of the ion and, subsequently, the empirical abundance of a given element (see, for example, Osterbrock 1989). In low density regions, such as those in H II galaxies, the permitted-line emissivities (like those of H and He) decrease (slowly) with T and are usually independent of n_e, unless collisional effects are

important. On the other hand, the forbidden-line emissivities (like those of O, N, S) are strongly dependent on T, and the dependence on the density may also be important. In H II galaxies, the [S II] line ratio indicates low electron densities ($n_e < 500$ cm^{-3}), and except for those with high temperature, the collisional effects are negligible. Thus, the derived heavy element abundances depend mainly on a good determination of the temperature. For these regions, the temperature used for the high-ionization lines is that obtained from the [O III] line ratio, T_{OIII}, while for the low-ionization lines the temperature is derived from T_{OIII} using results from photoionization models (see for example, PSTE).

A similar method is usually applied to planetary nebulae. In contrast to H II regions, for several PNe the Balmer temperature T_B (obtained from the observed Balmer discontinuity) is also determined. In many cases T_B is found to be lower than T_{OIII} (Peimbert 1971; Liu & Danziger 1993). As pointed out by Peimbert (1971), this discrepancy could be due to temperature fluctuations, which, however, are not reproduced by the standard photoionization models for these nebulae (Liu & Danziger 1993). If, indeed, the gas temperature is overestimated by T_{OIII}, with the true temperature given by T_B, the heavy element abundances derived from forbidden lines using T_{OIII} are underestimated (Viegas & Clegg 1994). Indeed, the abnormal chemical abundances inferred for some H II galaxies with WR features may result from just such an overestimate of the gas temperature (Esteban & Peimbert 1995). A well-observed giant H II region, NGC 2363, shows just such a discrepancy between the Paschen temperature and T_{OIII} in two knots (González-Delgado et al. 1994). For the H II galaxies used to derive the primordial helium abundance, such an underestimate of the true oxygen abundance may introduce a systematic bias into the inferred value of Y_P. Furthermore, temperature fluctuations also will have a direct effect on the helium abundance determined from the recombination lines (Esteban & Peimbert 1995). Although the ratio of He and H emissivities is very nearly independent of temperature, the helium-line intensities are modified from the pure recombination case by collisional excitation (Cox & Daltabuit 1971; Ferland 1986) which is temperature-sensitive. If the "true" H II region temperature has been overestimated, the collisional correction has been overestimated and the "true" helium abundance underestimated. This effect will be largest for the hottest, metal-poor H II regions. Thus the combined effects, which tend to increase the derived oxygen abundance in the metal-rich H II regions and the helium abundance in the metal-poor regions (see

Figure 1), will “tilt” the inferred Y versus O/H relation, flattening the slope and increasing the intercept, Y_P. In this paper we analyse these effects in an attempt to quantify the corresponding systematic uncertainty in Y_P.

2 The Method

If the Paschen or Balmer temperatures were available, the corrections described above would be straightforward to implement. Unfortunately, for the low-metallicity, extragalactic H II regions used to derive the primordial abundance of helium, there appears to be no data on the Balmer or Paschen temperatures. To estimate the magnitude of the possible corrections we have, therefore, adopted a Monte Carlo approach to quantifying the potential systematic uncertainty introduced into the determination of Y_P by uncertainty in the temperature of the H II region gas.

We begin by adopting a “fiducial” data set using 43 H II galaxies from PSTE, ITL, Skillman & Kennicutt (1993) and Skillman *et al.* (1994). The role of this fiducial set is merely to provide a comparison, in order to quantify the *changes* induced by temperature differences. For this reason we have not employed the unpublished data of Skillman et al. (in preparation, see OS and OSS) nor those H II regions listed in PSTE which they, themselves, did not observe. We follow ITL and eliminate those regions for which their data is noisy or otherwise uncertain. For each H II region in our fiducial data set we adopt the same algorithms used by these authors to derive the temperature and the fractional abundances in the low-ionization zones. The electron density is obtained from the [S II] line ratio and we use the Brocklehurst (1972) recombination coefficients for the He and H lines along with the collisional correction for the He lines from Clegg (1987). The fractional abundance of He^+ is the (unweighted) average of the abundances obtained from HeI λ4471, HeI λ5876 and HeI λ6678. Ignoring any statistical uncertainties, we use these fiducial abundances to find a “standard” linear fit to Y versus O/H (unweighted) in the absence of temperature fluctuations. We emphasize that we are not so much interested in the “best fit” Y versus O/H relation for our fiducial data set but, rather, the *differences* in the Y versus O/H relations between ΔT zero and nonzero ($\Delta T \equiv T_{OIII} - T$). In Figure 1 each point in our data set is shown in the Y versus O/H plane in the absence of temperature fluctuations ($\Delta T = 0$) and for $\Delta T = 2000$K, joined by a solid line. As anticipated, the higher metallicity regions tend to be cooler (see Figure

2). This has the effect that the derived oxygen abundance in high-metallicity regions is very sensitive to temperature fluctuations as is seen clearly in Figure 1.

For planetary nebulae the observed temperature difference $\Delta T = T_{OIII} - T_B$ is typically 2000K (Liu & Danziger 1993) with one PN having $\Delta T = 6000$K. For the giant H II region NGC 2363, $\Delta T \geq 3000$K (González-Delgado et al. 1994). Thus, we have recalculated the helium and oxygen abundances for each H II region with ΔT chosen from a distribution which ranges from zero to ΔT_{max}, according to a probability $P(\Delta T)$ to be described below; ΔT_{max} is varied in the range $500 \leq \Delta T_{max} \leq 4000$K. We repeat this procedure 10,000 times for each choice of ΔT_{max}. For most H II regions Y does not decrease with decreasing temperature as might have been expected from the effective recombination coefficients alone (Esteban & Peimbert 1995) provided we account for the collisional effect on the He lines; for H II galaxies with high gas temperature, the collisional effect dominates, leading to an increase of Y. As seen in Figure 1, the decrease in T leads to an increase in the oxygen abundance. For each of the 10,000 realizations (for each choice of ΔT_{max}) of our H II region data set we fit a linear Y versus O/H relation to derive Y_P and the slope $\Delta Y/\Delta Z$, where the heavy element mass fraction $Z \approx 20(O/H)$ (see PSTE).

3 Results: Y Versus Z

3.1 Full Data Set

The results for our full data set of 43 H II regions, selected to have $O/H \leq 1.5\times 10^{-4}$, are shown in Figures 3 – 5. As anticipated, the intercept, Y_P, is systematically increased by an amount which depends on ΔT_{max} (see Figures 3a & 5). This increase in the intercept of the Y versus Z relation is accompanied by a flattening of the slope (see Figures 3b & 5). To probe the sensitivity of our results to the adopted probability distribution of ΔT values, $P(\Delta T)$, we have considered three, simple distributions: flat, linearly increasing, linearly decreasing. The results for these three, different distributions are shown for $\Delta T_{max} = 4000$K in Figure 4. Clearly, the effects are closely similar for all choices; in our subsequent discussion (and, in Figures 3 & 5) we present results for the flat distribution ($P(\Delta T)$ independent of ΔT).

In Figure 5 is shown how the magnitude of the systematic offsets in slope and in-

tercept scale with ΔT_{max}. For the not unreasonable choice of ΔT_{max} = 2000K, the increase in the inferred value of Y_P is significant, comparable to some estimates of the upper bound to the systematic uncertainty in Y (OS; OSS). For larger values of ΔT_{max}, the systematic offset will be even larger. It might have been anticipated (see OSS) that such large systematic offsets would be accompanied by an increase in the dispersion of the abundances around the best fit Y versus Z relation. To test for this, for each realization we have computed σ, the variance of the residuals between the data (Y) and those values predicted by the corresponding fit for that realization (Y_{fit}). However, as seen in Figure 5, for temperature fluctuations this effect of added dispersion, if present at all, is very small compared, for example, to the typical uncertainty in the individual H II region Y determinations which are of order 0.010 (OSS). Thus, until there are observations of H II region temperatures determined from hydrogen lines, it is difficult to set a firm upper bound to the magnitude of the systematic offset in Y_P due to the possible presence of temperature fluctuations. It is, therefore, important to consider how best to analyze current data so as to minimize the potential importance of such temperature fluctuations.

3.2 Truncated Data Set

A significant contribution to the systematic offset in Y_P in the presence of temperature fluctuations comes from the flattening of the slope of the Y versus Z relation due to the large increase in oxygen abundance for the cooler, higher-metallicity H II regions (see Figures 1 & 2). Therefore, the uncertainty in Y_P derived from a fit to the trend of Y with O/H might be minimized if the data set is restricted to the very lowest metal abundance H II regions which are hotter. To explore this, for each realization of our Monte Carlos we identify the subset of H II regions with low-metallicity: O/H $\leq 0.9\times 10^{-4}$. For these subsets we fit linear Y versus Z relations and compare the slopes and intercepts (as well as σ) to those found for the corresponding low-metallicity subset in the absence of temperature fluctuations. The results are shown in Figures 6 & 7. As expected, the changes in slope and intercept for this low-metallicity set are much smaller. Indeed, the trend is even opposite that for our full data set: *lower* intercept, *higher* slope (although the effect is so small as to be only marginally significant). The lesson is clear. If we wish to minimize the uncertainty due to temperature fluctuations

in Y_P derived from a fit of Y versus Z, we should restrict our attention to the most metal-poor H II regions.

3.3 Lowest-Y H II Regions

Since it is generally accepted that the helium abundance has only increased since the big bang, an alternate approach to using H II regions observations to infer the primordial abundance is to compute the mean of Y for those regions with the lowest helium abundances; then, $Y_P \leq \langle Y \rangle$ (OS; OSS). This estimator is likely to be robust against the systematic shifts due to temperature fluctuations. To test this we have first chosen the ten lowest-Y H II regions in our fiducial data set ($\Delta T = 0$) and found the unweighted average of Y, $\langle Y \rangle_{10}(0)$. Then, from our Monte Carlos, for various choices of ΔT_{max} we determine (for each realization) the ten lowest-Y H II regions (often, but not always, the same as in the fiducial set) and compute $\Delta\langle Y \rangle_{10} \equiv \langle Y \rangle_{10}(\Delta T) - \langle Y \rangle_{10}(0)$. The distributions of $\Delta\langle Y \rangle_{10}$ for ΔT_{max} = 2000K and 4000K are shown in Figure 8 and the trend of $\Delta\langle Y \rangle_{10}$ with ΔT_{max} is shown in Figure 9 where the corresponding change in the dispersion around the mean (σ) is also shown. As expected, although temperature fluctuations tend to increase $\langle Y \rangle_{10}$ systematically (mainly due to the reduced correction for collisional excitation), the effect is quite small.

4 Discussion

Temperature fluctuations in the low-metallicity, extragalactic H II regions used to infer the primordial helium abundance will lead to systematic offsets in the helium and oxygen abundances derived for those regions. In the absence of direct observations of the hydrogen temperatures for these regions we have attempted to quantify the effect of such fluctuations on the derived value of Y_P by performing a series of Monte Carlo calculations where the temperature fluctuation for each H II region is chosen from a distribution whose maximum value is varied. Y_P determined from a linear fit to Y versus O/H is especially susceptible to this source of error which flattens the Y versus Z relation resulting in a higher intercept. This potential systematic error is insidious in the sense that the offset in Y_P may be significant without a noticeable increase in the dispersion of the individual H II region Y-values around the best fit Y versus Z relation. To illustrate the possible offsets which may be consistent with extant data, consider the results of

OSS for their full data set of 62 H II regions with O/H $\leq 1.5\times10^{-4}$: $Y_P = 0.234\pm0.005$ (0.005 is a 2-σ estimate of the statistical uncertainty). For ΔT_{max} = 4000K we should increase this value by 0.008 ± 0.004 (see Figure 5). Adding the (uncorrelated) systematic and statistical uncertainties in quadrature will lead us to $Y_P = 0.242\pm0.006$ (95%CL). Such a large correction to Y_P would ameliorate the "tension" in BBN (Hata et al. 1995) between primordial helium and the low abundance of deuterium suggested by Galactic observations (see, e.g., Dearborn, Steigman & Tosi 1995 and references therein) and by some direct detections of deuterium in low-metallicity, high-redshift QSO absorbers (Tytler, Fan & Burles 1996; Burles & Tytler 1996).

Until data on temperature fluctuations in these key extragalactic H II regions become available, the best strategy is to analyze the current data in a manner which minimizes this potential systematic error. One possibility is to restrict attention to the lowest-metallicity regions available. OS and OSS have done this for the subset with O/H $\leq$ 0.9×10^{-4} for which OSS derive from a linear fit to Y versus O/H: $Y_P = 0.230\pm0.007$ (95%CL). From our Monte Carlos we find for this subset that the systematic correction is likely very small (indeed, it may even be negative!); for ΔT_{max} = 4000K, ΔY_P = -0.004 ± 0.005 (see Figure 7). If, perhaps naively, we apply this correction to the OSS result, we infer: $Y_P = 0.226\pm0.009$ (95%CL). In this case the "tension" between primordial helium and low primordial deuterium is exacerbated.

Another "safe" approach to using the existing data to derive an estimate of Y_P is to ignore the metallicity information and simply take a mean ($\langle Y \rangle$) of the helium abundance for the lowest abundance H II regions. Since helium is only expected to increase after BBN, this provides an upper bound to Y_P. In general, the H II regions with the lowest values of Y tend to be the lowest-metallicity regions which are also the hottest (see Figure 2). For such regions there has often been a non-negligible correction for collisions in deriving Y from the helium line intensities. If the gas is, in fact, cooler, this correction has been overestimated and the "true" value of Y should be larger. Thus, temperature fluctuations will increase $\langle Y \rangle$. From our Monte Carlos we have selected, for each realization, the ten H II regions with the lowest Y-values and we have found, for ΔT_{max} = 4000K, a small systematic increase: $\Delta\langle Y \rangle_{10} = 0.002\pm0.001$ (see Figures 8 & 9). For their ten lowest-Y regions, OSS find $\langle Y \rangle_{10} = 0.230\pm0.006$ (95%CL), so that even with our largest correction we infer a revised mean of 0.232 ± 0.006 suggesting that $Y_P \leq 0.238$ (95%CL). Here, too, the "tension" between D and ^{4}He fails to be relieved.

5 Summary

Temperature fluctuations in low-metallicity, extragalactic H II regions may have a significant effect on the determination of the primordial abundance of helium. If present, they may increase the metallicity of the higher-metallicity, relatively cooler regions and increase the helium abundance of the more metal-poor, hotter regions, tilting the inferred Y versus Z relation and leading to a higher, zero-metallicity intercept (Y_P). Such a systematic offset is not necessarily accompanied by a significant increase in the dispersion of the data around the best fit linear Y versus Z relation and, therefore, may remain invisible in the absence of direct data on temperature differences in these regions. It is clear that such data is crucial to constraining the uncertainty in Y_P. In the absence of such data, we have noted that restricting attention to the lowest-metallicity regions ($Z \leq Z_\odot/10$) will tend to minimize this systematic offset. Alternatively, the mean of the helium abundances of the lowest-Y H II regions is also robust against the effect of temperature fluctuations.

Acknowledgments

S.M.V. and R.G. owe a debt of gratitude to P. Benevides-Soares for valuable discussions. The work of S.M.V. and R.G. is partially supported by FAPESP and CNPq in Brasil; that of G.S. is supported by the DOE. This work was initiated when S.M.V. and G.S. were visiting the INT at the University of Washington and they are grateful for the hospitality and stimulating environment provided there.

References

Brocklehurst, M. 1972, MNRAS, 157, 211

Burles, S. & Tytler, D. 1996, ApJ, 460, 584

Clegg, R.E.S. 1987, MNRAS, 229, 31P

Cox, D.P. & Daltabuit, E. 1971, ApJ, 167, 257

Davidson, K. & Kinman, T.D. 1985, ApJS, 58, 321

Esteban, C. & Peimbert, M. 1995, A&A, 300, 78

Ferland, G.J. 1986, ApJ, 310, L67

González-Delgado, R.M., Pérez, E., Tenorio-Tagle, G., Vilchez, J. M., Terlevich, E., Terlevich, R., Telles, E., Rodriguez-Espinosa, J. M., Mas-Hesse, M., García-Vargas, M. L., Díaz, A. I., Cepa, J. & Castaněda , J. 1994, ApJ 437, 239

Hata, N., Scherrer, R.J., Steigman, G., Thomas, D., Walker, T.P., Bludman, S. & Langacker, P. 1995, Phys. Rev. Lett., 75, 3977

Izotov, Y.I., Thuan, T.X., & Lipovetsky, V.A. 1994 ApJ 435, 647. (ITL)

Izotov, Y.I., Thuan, T.X., & Lipovetsky, V.A. 1997, ApJS, 108, 1 (ITL)

Liu, X. & Danziger, J. 1993, MNRAS, 263, 256

Olive, K.A., Skillman, E. & Steigman, G. 1997, ApJ, In Press (OSS)

Olive, K.A., & Steigman, G. 1995, ApJS, 97, 49 (OS)

Osterbrock, D.E. 1989, Astrophysics of Gaseous Nebulae and Active Galactic Nuclei, University Science Books, Mill Valley, California

Pagel, B E.J., Simonson, E.A., Terlevich, R.J. & Edmunds, M. 1992, MNRAS, 255, 325 (PSTE)

Peimbert, M. 1971, Bol. Obs. Tonantzintla y Tacubaya, 6, 29

Peimbert, M. 1996, Rev. Mex. Astr. Astrofis., Serie de Conferencias, 4, 55

Peimbert, M. & Costero, R. 1969, Bol. Obs. Tonantzintla y Tacubaya, 5, 3

Skillman, E., & Kennicutt, R.C. 1993, ApJ, 411, 655

Skillman, E., Terlevich, R.J., Kennicutt, R.C., Garnett, D.R., & Terlevich, E. 1994, ApJ, 431, 172

Steigman, G., Schramm, D.N. & Gunn, J. 1977, Phys. Lett, B66, 202

Tytler, D., Fan, X.-M., and Burles, S. 1996, Nature, 381, 207

Viegas, S.M. & Clegg, R. 1994, MNRAS, 271, 993

Figure Captions

Figure 1: The effect of temperature fluctuations on the helium and oxygen abundances for our fiducial data set. The left tick mark is for no temperature fluctuations ($\Delta T = 0$) and the right tick mark is for $\Delta T = 2000K$.

Figure 2: The temperatures of the H II regions in our fiducial data set as a function of the oxygen abundance (evaluated for $\Delta T = 0$).

Figure 3a: The distribution of offsets in the primordial mass fraction of ^{4}He (inferred from a linear fit to Y versus O/H) due to temperature fluctuations for $\Delta T_{max} = 2000K$ (shaded histogram) and $\Delta T_{max} = 4000K$. The results here are for the full data set ($O/H \leq 1.5{\times}10^{-4}$).

Figure 3b: The corresponding distribution of the ratios of the slopes of the Y versus Z ($Z \approx 20(O/H)$) relations for $\Delta T_{max} = 2000K$ (shaded histogram) and $\Delta T_{max} = 4000K$ compared to the fiducial slope (for $\Delta T = 0$).

Figure 4: The distribution of offsets in the primordial mass fraction of ^{4}He evaluated for the full data set with $\Delta T_{max} = 4000K$ for three different temperature probability distributions (from top to bottom: flat, linearly increasing, linearly decreasing).

Figure 5: In the three panels the solid curves show the variation of the mean values of the offsets in Y_P (top panel), the ratio of slopes (middle panel) and the dispersion around the best fit linear Y vs. Z relation (bottom panel) as a function of ΔT_{max}. The dotted curves show the 95%CL ranges.

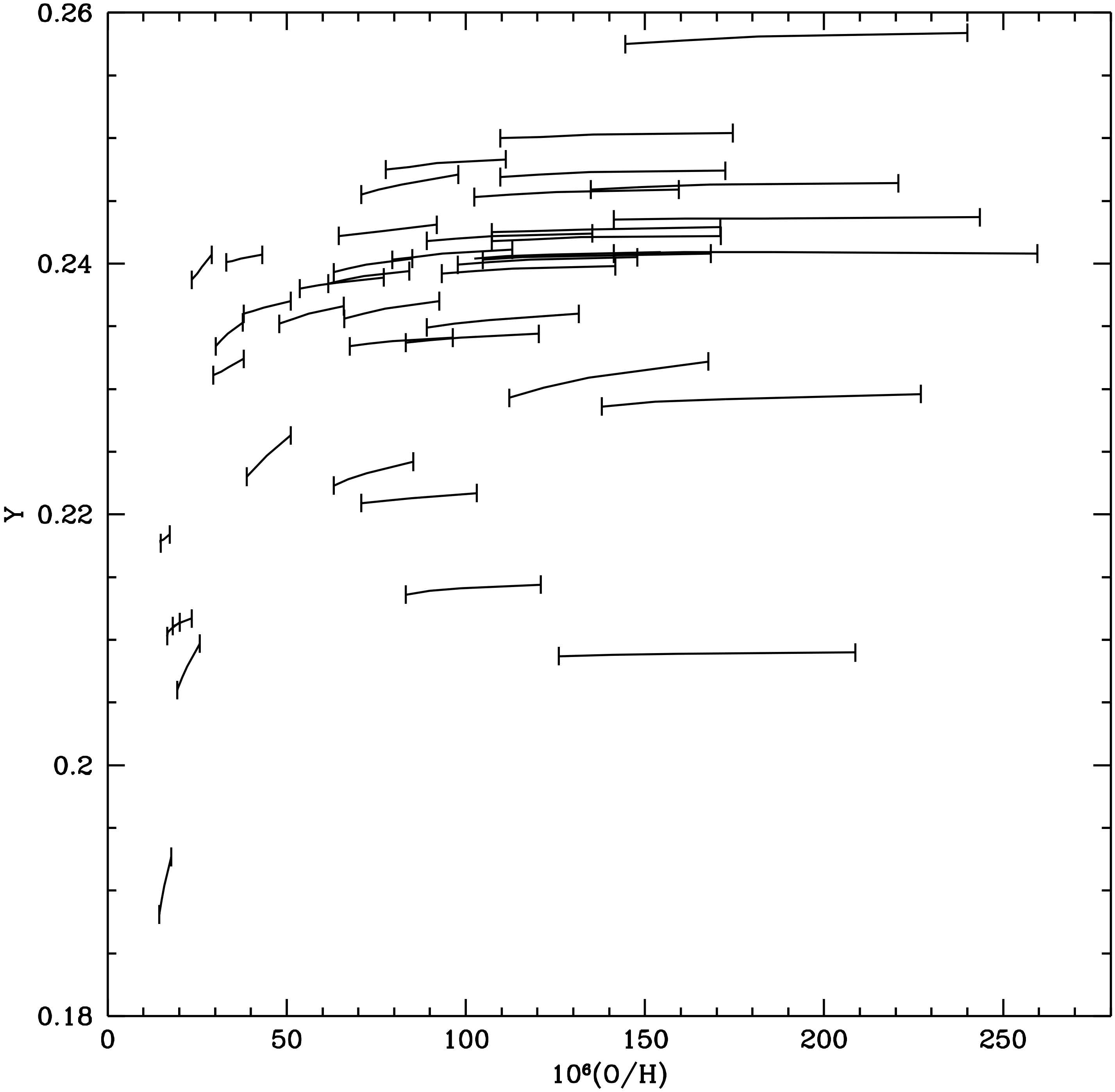
Figure 6a: Similar to Figure 3a, but for the low-metallicity subset of H II regions ($O/H \leq 0.9{\times}10^{-4}$).

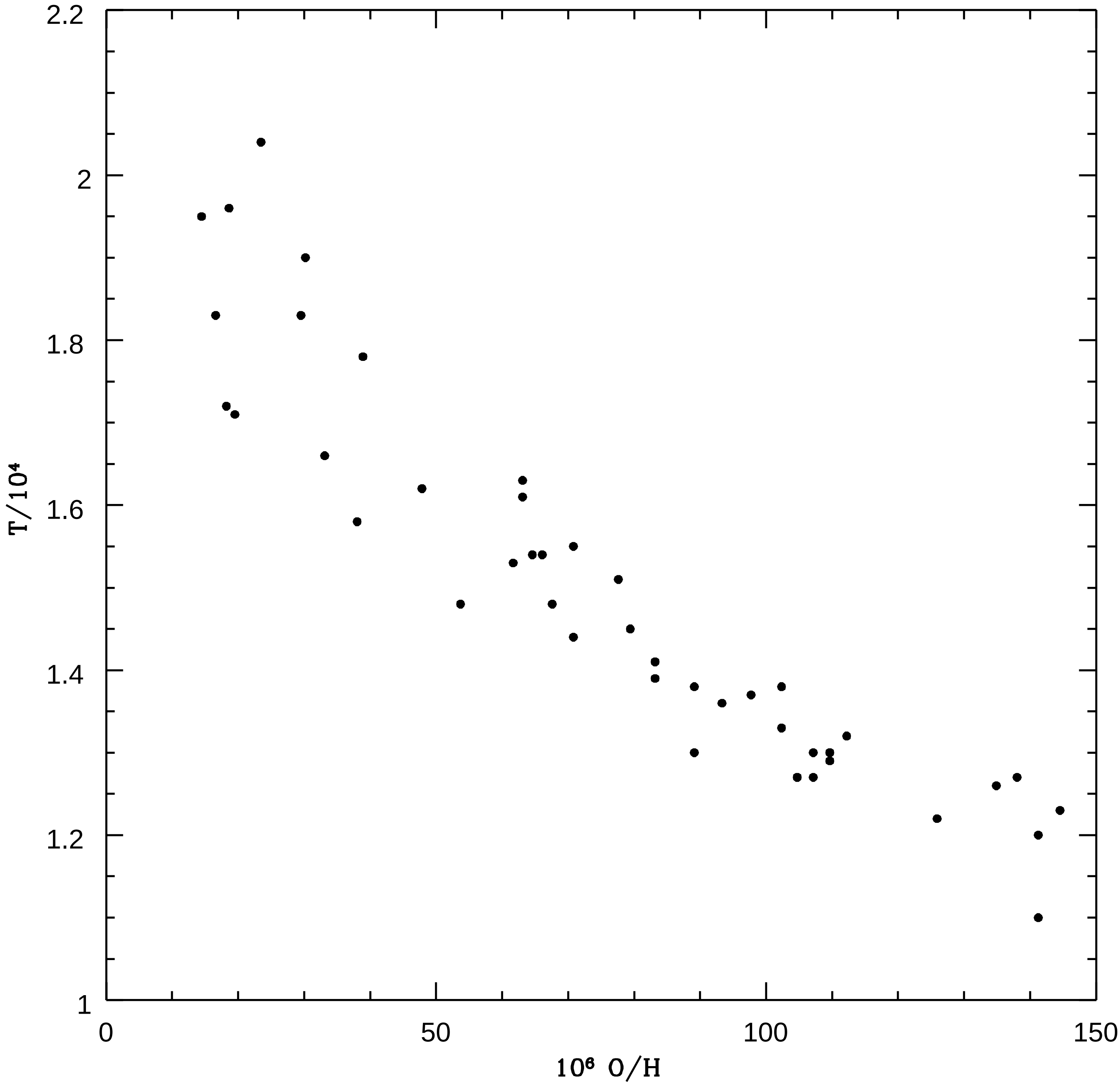
Figure 6b: Similar to Figure 3b, but for the low-metallicity subset of H II regions.

Figure 7: Similar to Figure 5, but for the low-metallicity subset of H II regions ($O/H \leq 0.9{\times}10^{-4}$).

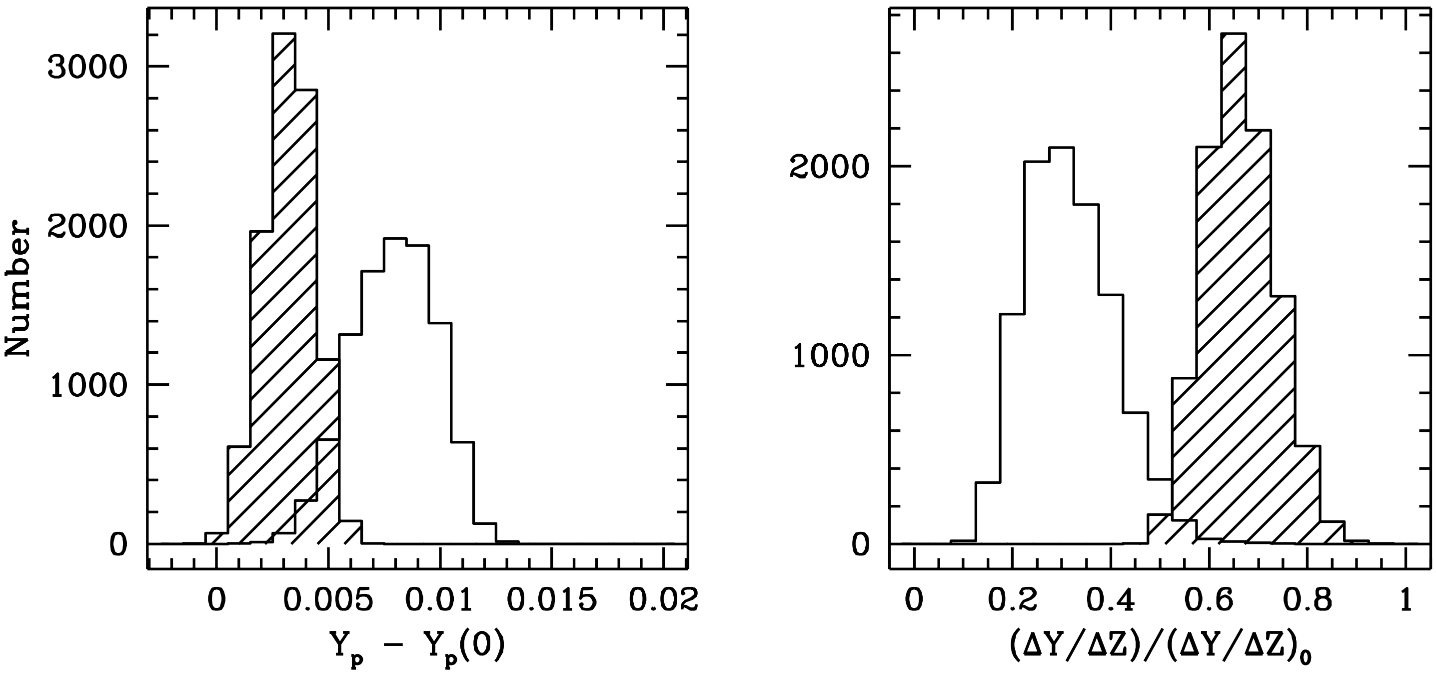
Figure 8: The distribution of offsets in the means of the ten lowest-Y H II region helium abundances for $\Delta T_{max} = 2000K$ (shaded histogram) and $\Delta T_{max} = 4000K$.

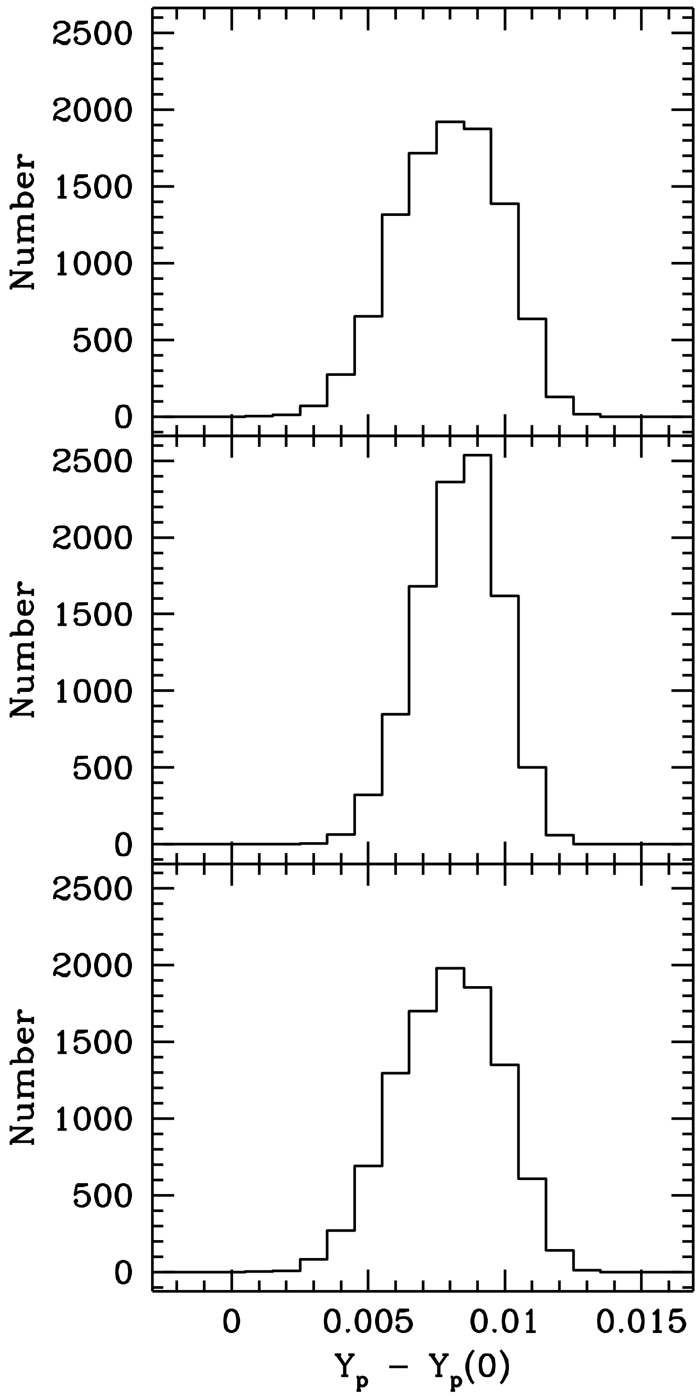
Figure 9: Similar to Figures 5 & 7 for the means of the ten lowest-Y H II region helium abundances as a function of ΔT_{max}.


0.26
0.24
0.22
0.2
0.18
Y
0
50
100
150
200
250
10^6(O/H)

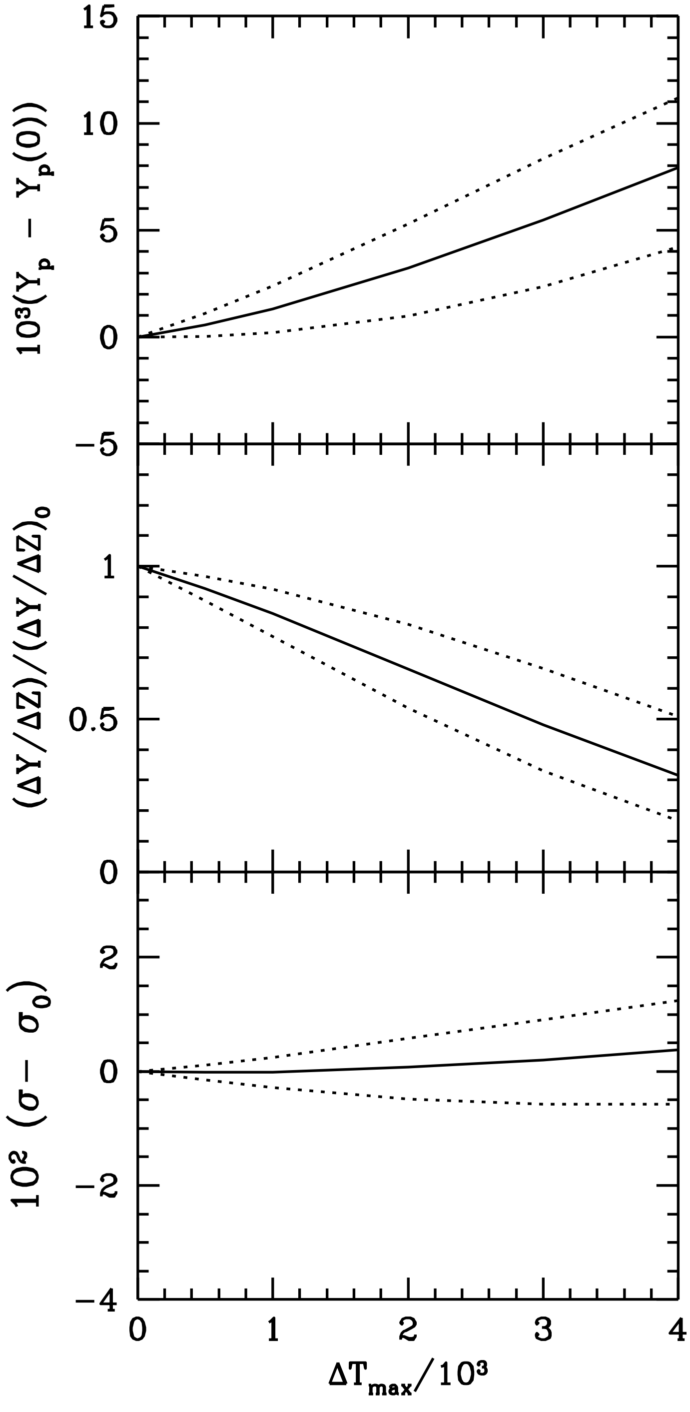


2.2
2
1.8
1.6
1.4
1.2
1
T/10⁴
0
50
100
150
10⁶ O/H

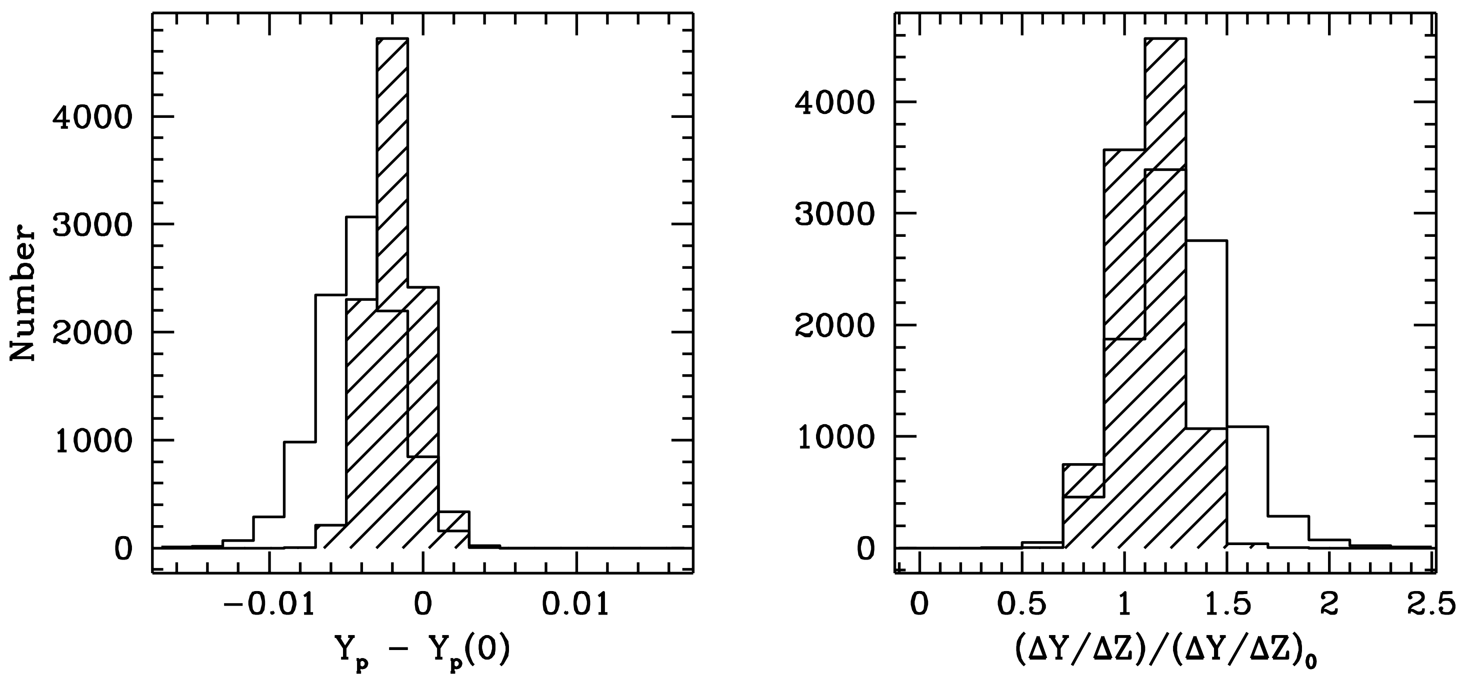

Number
3000
2000
1000
0
0
0.005
0.01
0.015
0.02
$Y_p - Y_p(0)$
2000
1000
0
0
0.2
0.4
0.6
0.8
1
$(\Delta Y/\Delta Z)/(\Delta Y/\Delta Z)_0$



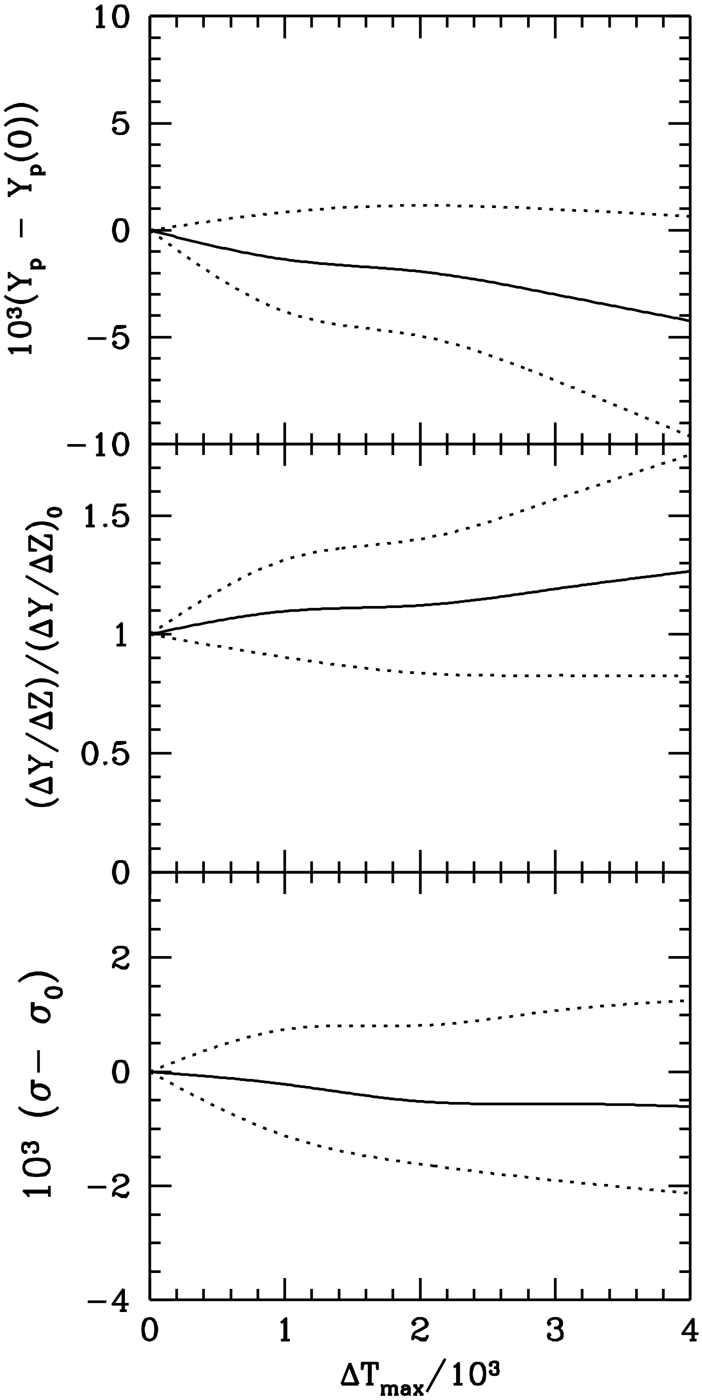
2500
2000
1500
1000
500
0
Number
0
0.005
0.01
0.015
$Y_p - Y_p(0)$



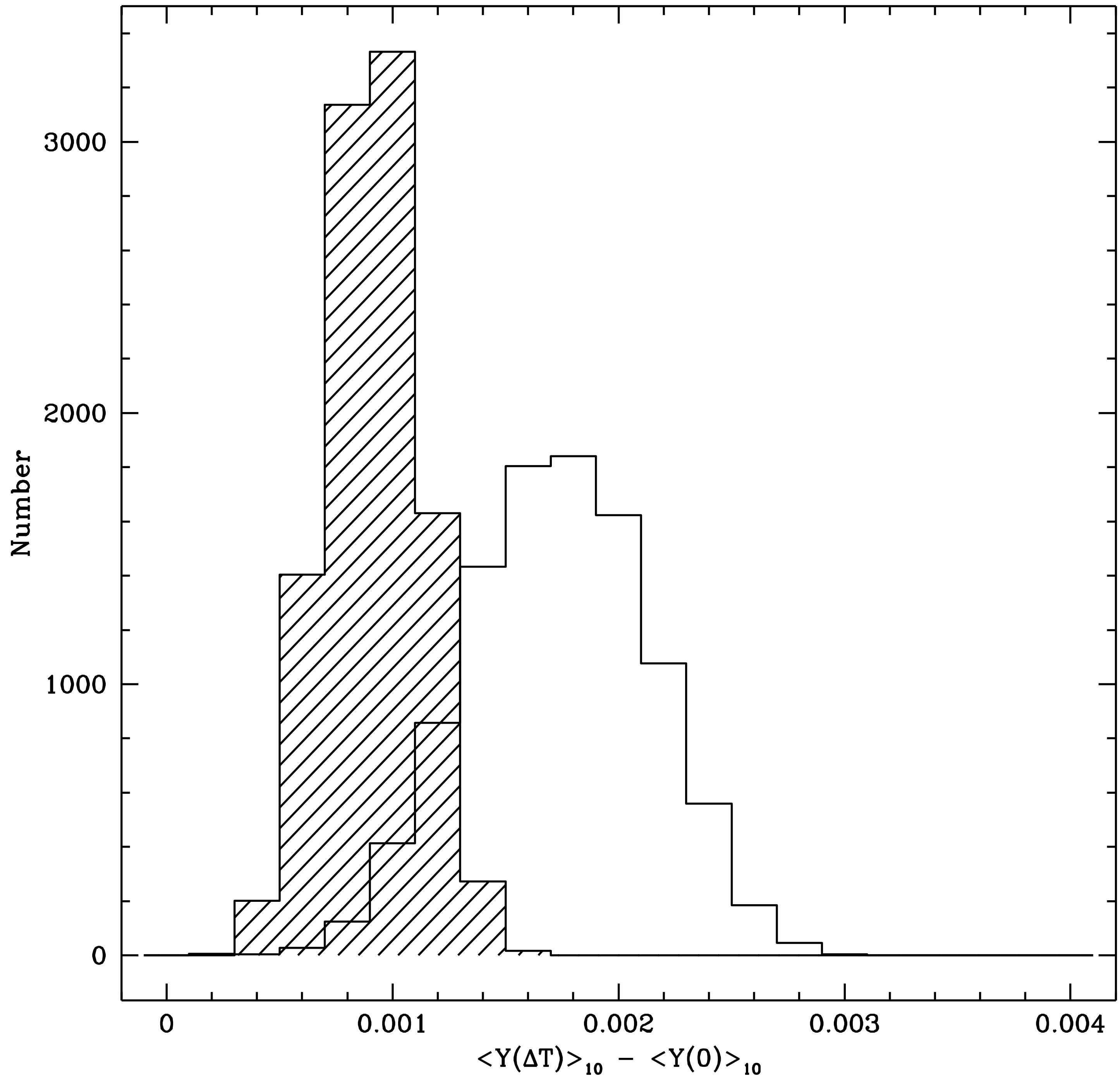
15
10
5
0
−5
1
0.5
0
2
0
−2
−4
0
1
2
3
4
$10^3(Y_p - Y_p(0))$
$(\Delta Y/\Delta Z)/(\Delta Y/\Delta Z)_0$
$10^2\ (\sigma - \sigma_0)$
$\Delta T_{max}/10^3$

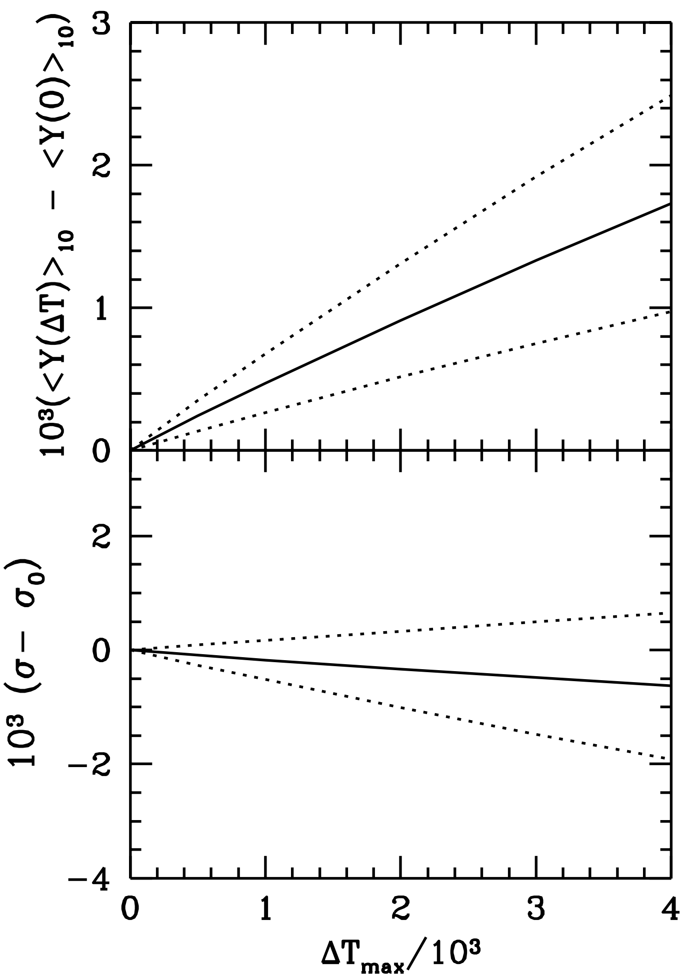


Number
4000
3000
2000
1000
0
−0.01
0
0.01
$Y_p - Y_p(0)$
4000
3000
2000
1000
0
0
0.5
1
1.5
2
2.5
$(\Delta Y/\Delta Z)/(\Delta Y/\Delta Z)_0$



$10^3(Y_p - Y_p(0))$
$(\Delta Y/\Delta Z)/(\Delta Y/\Delta Z)_0$
$10^3\ (\sigma - \sigma_0)$
$\Delta T_{max}/10^3$


3000
2000
1000
0
Number
0
0.001
0.002
0.003
0.004
<Y(ΔT)>10 − <Y(0)>10



$10^3(<Y(\Delta T)>_{10} - <Y(0)>_{10})$
$10^3\ (\sigma - \sigma_0)$
$\Delta T_{max}/10^3$
0
1
2
3
4
−2
−4